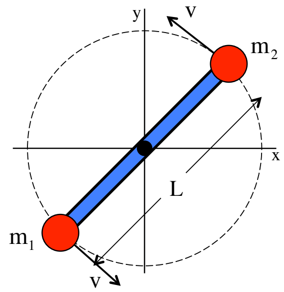
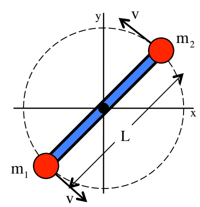
Problem 8.45



A light rod of length *L = 100 m* rotates about an axis perpendicular to its length and passing through its center as shown. Two particles of mass $m_1 = 4\text{ kg}$ and $m_2 = 3\text{ kg}$ are connected to the end of a light rod.

a.) what is the system's kinetic energy when $\omega = 2.5\text{ rad/sec}$?

b.) what would the system's kinetic energy be if the rod's mass was 2 kg?

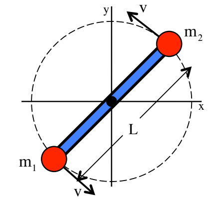


A light rod of length *L = 100 m* rotates about an axis perpendicular to its length and passing through its center as shown. Two particles of mass $m_1 = 4\text{ kg}$ and $m_2 = 3\text{ kg}$ are connected to the end of a light rod.

a.) what is the system's kinetic energy when $\omega = 2.5\text{ rad/sec}$?

Interesting, you can do this two ways. Looking at the particles as point masses rotating in a circle of radius L/2:

$$\begin{aligned}KE &= \frac{1}{2} I_{ptmass}\omega^2 + \frac{1}{2} I_{ptmass}\omega^2\\ &= \frac{1}{2}\left(m_1 r^2\right)\omega^2 + \frac{1}{2}\left(m_2 r^2\right)\omega^2\\ &= \frac{1}{2}\left(m_1\left(\frac{L}{2}\right)^2\right)\omega^2 + \frac{1}{2}\left(m_2\left(\frac{L}{2}\right)^2\right)\omega^2\\ &= \frac{1}{2}\left((4\text{ kg})\left(\frac{1\text{ m}}{2}\right)^2\right)(2.5\text{ rad/s})^2 + \frac{1}{2}\left((4\text{ kg})\left(\frac{1\text{ m}}{2}\right)^2\right)(2.5\text{ rad/s})^2\\ &= 6.25\text{ joules}\end{aligned}$$

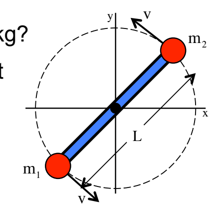


a.) The other way? Looking at the particles as point masses moving with velocity magnitude v, where v is:

$$\begin{aligned}v &= r\omega\\ &= (.5\text{ m/rad})(2.5\text{ rad/sec})\\ &= 1.25\text{ m/sec}\end{aligned}$$

so

$$\begin{aligned}KE &= \frac{1}{2} m_1 v^2 + \frac{1}{2} m_2 v^2\\ &= \frac{1}{2}\left(m_1 [r\omega]^2\right) + \frac{1}{2}\left(m_1 [r\omega]^2\right)\\ &= \frac{1}{2}\left((4\text{ kg})[(.5\text{ m})(2.5\text{ rad/s})]^2\right) + \frac{1}{2}\left((4\text{ kg})[(.5\text{ m})(2.5\text{ rad/s})]^2\right)\\ &= 6.25\text{ joules}\end{aligned}$$



b.) what is the system's kinetic energy when the rod's mass is 2 kg?

Again, there are two ways. I'll only use one. The point is that you have to include the KE of the massive rod. Treating the masses as point masses moving with velocity v, and remembering that

$$\begin{aligned}v &= r\omega\\ &= (.5\text{ m/rad})(2.5\text{ rad/sec})\\ &= 1.25\text{ m/sec}\end{aligned}$$

$$\begin{aligned}KE &= \frac{1}{2}m_1v^2 + \frac{1}{2}m_2v^2 + \frac{1}{2}I_{bar}\omega^2\\ &= 2\left(\frac{1}{2} m_1 v^2\right) + \frac{1}{2}\left[\frac{1}{12}(m_{bar})\left(\frac{L}{2}\right)^2\right]\omega^2\\ &= 2\left(\frac{1}{2}(4\text{ kg})[(1.25\text{ m/s})]^2\right) + \frac{1}{2}\left[\frac{1}{12}(2\text{ kg})\left(\frac{1\text{ m}}{2}\right)^2\right][(2.5\text{ m/s})]^2\\ &= 6.38\text{ joules}\end{aligned}$$